Novelty Journals

ISSN 2394-7314

International Journal of Novel Research in Computer Science and Software Engineering
Vol. 10, Issue 1, pp: (18-21), Month: January - April 2023, Available at: www.noveltyjournals.com

A Study on a Fractional Algebraic Problem

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DOI: https://doi.org/10.5281/zenodo.7605039

Published Date: 04-February-2023

***Abstract*: In this paper, we study a fractional algebraic problem based on a new multiplication of fractional analytic functions. We can solve this fractional algebraic problem by using some techniques. In fact, the solutions are generalizations of the solutions of ordinary algebraic problem.**

***Keywords*: Fractional algebraic problem, New multiplication, Fractional analytic functions.**

I. INTRODUCTION

Fractional calculus is a branch of mathematical analysis which deals with the research and applications of integrals and derivatives of arbitrary order. In recent decades, the applications of fractional calculus in various fields of science is growing rapidly, such as applied mathematics, physics, bioengineering, mechanics, control theory, electrical engineering, modelling, financial economics, etc [1-12].

In this paper, based on a new multiplication of fractional analytic functions, a fractional algebraic problem is studied. Using some methods, the solutions of this fractional algebraic problem can be obtained. On the other hand, our results are generalizations of the results of classical algebraic problem.

II. PRELIMINARIES

Definition 2.1 ([13])**:** Assume that x and a_k are real numbers for all k, and $0 < \alpha \le 1$. If the function $f_\alpha: [a, b] \to R$ can be expressed as an α-fractional power series, that is, $f_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)} x^{k\alpha}$, then we say that $f_\alpha(x^\alpha)$ is α-fractional analytic function.

In the following, we introduce a new multiplication of fractional analytic functions.

Definition 2.2 ([14])**:** If $0 < \alpha \le 1$. Suppose that $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ are two α-fractional analytic functions,

$$f_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)} x^{k\alpha} = \sum_{k=0}^{\infty} \frac{a_k}{k!} \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes k}, \tag{1}$$

$$g_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{b_k}{\Gamma(k\alpha+1)} x^{k\alpha} = \sum_{k=0}^{\infty} \frac{b_k}{k!} \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes k}. \tag{2}$$

Then

$$\begin{aligned} & f_\alpha(x^\alpha) \otimes g_\alpha(x^\alpha) \\ & = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)} x^{k\alpha} \otimes \sum_{k=0}^{\infty} \frac{b_k}{\Gamma(k\alpha+1)} x^{k\alpha} \\ & = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)} \left(\sum_{m=0}^{k} \binom{k}{m} a_{k-m} b_m\right) x^{k\alpha}. \end{aligned} \tag{3}$$

Equivalently,

$$f_\alpha(x^\alpha)\otimes g_\alpha(x^\alpha)$$
$$=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}\otimes\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}$$
$$=\sum_{k=0}^{\infty}\frac{1}{k!}\left(\sum_{m=0}^{k}\binom{k}{m}a_{k-m}b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}. \quad (4)$$

Definition 2.3 ([15])**:** If $0<\alpha\leq 1$, and $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ are two α-fractional analytic functions. Then $\left(f_\alpha(x^\alpha)\right)^{\otimes n}=f_\alpha(x^\alpha)\otimes\cdots\otimes f_\alpha(x^\alpha)$ is called the nth power of $f_\alpha(x^\alpha)$. On the other hand, if $f_\alpha(x^\alpha)\otimes g_\alpha(x^\alpha)=1$, then $g_\alpha(x^\alpha)$ is called the $\otimes$ reciprocal of $f_\alpha(x^\alpha)$, and is denoted by $\left(f_\alpha(x^\alpha)\right)^{\otimes -1}$.

III. MAIN RESULTS

In this section, we solve a fractional algebraic problem. At first, we need a lemma.

Lemma 3.1: *Suppose that* $0<\alpha\leq 1$, p,t *are real numbers, and* n *is a positive integer. If* $\frac{1}{\Gamma(\alpha+1)}t^\alpha=p$, *then*

$$\left[\frac{1}{\Gamma(\alpha+1)}t^\alpha\right]^{\otimes n}=\frac{n!\cdot[\Gamma(\alpha+1)]^n}{\Gamma(n\alpha+1)}\cdot p^n \quad (5)$$

Proof Since $t^\alpha=\Gamma(\alpha+1)\cdot p$, it follows that

$$\left[\frac{1}{\Gamma(\alpha+1)}t^\alpha\right]^{\otimes n}$$
$$=\frac{n!}{\Gamma(n\alpha+1)}t^{n\alpha}$$
$$=\frac{n!}{\Gamma(n\alpha+1)}(t^\alpha)^n$$
$$=\frac{n!}{\Gamma(n\alpha+1)}[\Gamma(\alpha+1)\cdot p]^n$$
$$=\frac{n!\cdot[\Gamma(\alpha+1)]^n}{\Gamma(n\alpha+1)}\cdot p^n .$$

Q.e.d.

Problem 3.2: Assume that $0<\alpha\leq 1$, and λ is a real number, $\lambda\neq 0$. If

$$\frac{1}{\Gamma(\alpha+1)}x^\alpha\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 2}-\frac{1}{\Gamma(\alpha+1)}x^\alpha+1\right]^{\otimes -1}=\lambda . \quad (6)$$

Find

$$\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 2}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 4}-\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 2}+1\right]^{\otimes -1} , \quad (7)$$

and

$$\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 3}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 6}-\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 3}+1\right]^{\otimes -1} . \quad (8)$$

Solution Since

$$\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes -1}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes 2}-\frac{1}{\Gamma(\alpha+1)}x^\alpha+1\right]=\frac{1}{\lambda} . \quad (9)$$

It follows that

$$\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]+\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha\right]^{\otimes -1}=\frac{1}{\lambda}+1 . \quad (10)$$

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Let

$$\left[\frac{1}{\Gamma(\alpha+1)}r^{\alpha}\right]^{\otimes -1}=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 4}-\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+1\right]^{\otimes -1}. \tag{11}$$

Then

$$\begin{aligned}&\frac{1}{\Gamma(\alpha+1)}r^{\alpha}\\&=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -2}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 4}-\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+1\right]\\&=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -2}-1\\&=\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]+\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -1}\right]^{\otimes 2}-3\\&=\frac{2\cdot[\Gamma(\alpha+1)]^2}{\Gamma(2\alpha+1)}\cdot\left(\frac{1}{\lambda}+1\right)^2-3\,. \qquad \text{(by Lemma 3.1)}\end{aligned} \tag{12}$$

Therefore,

$$\begin{aligned}&\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 4}-\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 2}+1\right]^{\otimes -1}\\&=\left[\frac{1}{\Gamma(\alpha+1)}t^{\alpha}\right]^{\otimes -1}\\&=\frac{1}{\frac{2\cdot[\Gamma(\alpha+1)]^2}{\Gamma(2\alpha+1)}\cdot\left(\frac{1}{\lambda}+1\right)^2-3}\,.\end{aligned} \tag{13}$$

On the other hand, let

$$\left[\frac{1}{\Gamma(\alpha+1)}s^{\alpha}\right]^{\otimes -1}=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 6}-\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}+1\right]^{\otimes -1}. \tag{14}$$

Then by Lemma 3.1,

$$\begin{aligned}&\frac{1}{\Gamma(\alpha+1)}s^{\alpha}\\&=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -3}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 6}-\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}+1\right]\\&=\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}+\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -3}-1\\&=\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]+\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -1}\right]^{\otimes 3}-3\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]+\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes -1}\right]-1\\&=\frac{6\cdot[\Gamma(\alpha+1)]^3}{\Gamma(3\alpha+1)}\cdot\left(\frac{1}{\lambda}+1\right)^3-3\left(\frac{1}{\lambda}+1\right)-1.\end{aligned} \tag{15}$$

Thus,

$$\begin{aligned}&\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}\otimes\left[\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 6}-\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3}+1\right]^{\otimes -1}\\&=\left[\frac{1}{\Gamma(\alpha+1)}s^{\alpha}\right]^{\otimes -1}\\&=\frac{1}{\frac{6\cdot[\Gamma(\alpha+1)]^3}{\Gamma(3\alpha+1)}\cdot\left(\frac{1}{\lambda}+1\right)^3-3\left(\frac{1}{\lambda}+1\right)-1}\,.\end{aligned} \tag{16}$$

IV. CONCLUSION

In this paper, based on a new multiplication of fractional analytic functions, we study a fractional algebraic problem. In fact, the fractional algebraic problem is a generalization of traditional algebraic problem. By some methods, we can find the solutions of this fractional algebraic problem. In the future, we will continue to use the new multiplication of fractional analytic functions to solve the problems in applied mathematics and fractional differential equations.

REFERENCES

[1] Mohd. Farman Ali, Manoj Sharma, Renu Jain, An application of fractional calculus in electrical engineering, Advanced Engineering Technology and Application, vol. 5, no. 2, pp, 41-45, 2016.

[2] H. A. Fallahgoul, S. M. Focardi and F. J. Fabozzi, Fractional calculus and fractional processes with applications to financial economics, theory and application, Elsevier Science and Technology, 2016.

[3] J. T. Machado, Fractional Calculus: Application in Modeling and Control, Springer New York, 2013.

[4] R. L. Magin, Fractional calculus in bioengineering, 13th International Carpathian Control Conference, 2012.

[5] E. Soczkiewicz, Application of fractional calculus in the theory of viscoelasticity, Molecular and Quantum Acoustics, vol.23, pp.397-404, 2002.

[6] M. F. Silva, J. A. T. Machado, and I. S. Jesus, Modelling and simulation of walking robots with 3 dof legs, in Proceedings of the 25th IASTED International Conference on Modelling, Identification and Control (MIC '06), pp. 271-276, Lanzarote, Spain, 2006.

[7] M. Teodor, Atanacković, Stevan Pilipović, Bogoljub Stanković, Dušan Zorica, Fractional Calculus with Applications in Mechanics: Vibrations and Diffusion Processes, John Wiley & Sons, Inc., 2014.

[8] C. -H. Yu, A study on fractional RLC circuit, International Research Journal of Engineering and Technology, vol. 7, no. 8, pp. 3422-3425, 2020.

[9] C. -H. Yu, A new insight into fractional logistic equation, International Journal of Engineering Research and Reviews, vol. 9, no. 2, pp.13-17, 2021.

[10] R. Hilfer (Ed.), Applications of Fractional Calculus in Physics, WSPC, Singapore, 2000.

[11] F. Duarte and J. A. T. Machado, Chaotic phenomena and fractional-order dynamics in the trajectory control of redundant manipulators, Nonlinear Dynamics, vol. 29, no. 1-4, pp. 315-342, 2002.

[12] F. Mainardi, Fractional Calculus: Theory and Applications, Mathematics, vol. 6, no. 9, 145, 2018.

[13] C. -H. Yu, Study of fractional analytic functions and local fractional calculus, International Journal of Scientific Research in Science, Engineering and Technology, vol. 8, no. 5, pp. 39-46, 2021.

[14] C. -H. Yu, Solution of some type of improper fractional integral, International Journal of Interdisciplinary Research and Innovations, vol. 11, no. 1, pp. 11-16, 2023.

[15] C. -H. Yu, Research on a fractional exponential equation, International Journal of Novel Research in Interdisciplinary Studies, vol. 10, no. 1, pp. 1-5, 2023.